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DEVELOPMENT OF THERMOGRAVITATIONAL CONVECTION IN THE PRESENCE OF SOLUBLE SURFACE-ACTIVE MATTER AT THE INTERFACE

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The effect of insoluble surface-active matter (SAM), deposited on the interface, on the development of thermogravitational convection in a two-layer system was investigated in [1], where a new type of oscillatory instability of the equilibrium was detected.

The present article explores the effect of SAM solubility on the monotonic and oscillatory modes of instability. It has been found that an increase in SAM solubility lowers the monotonic instability threshold and narrows the range of parameters where oscillatory instability is more critical.

Assume that the space between two solid horizontal plates, which are maintained at different constant temperatures (the temperature difference is θ), is filled with two viscous, immiscible mediums. The coordinate origin is located at the interface, the x axis is horizontal, and the y axis points vertically upward. The equations of the solid boundaries are $y = a_1$ and $y = -a_2$. The coefficients of dynamic and kinematic viscosity, thermal conductivity, thermal diffusivity, and volume expansion are η_m, ν_m, $\varkappa_m$, χ_m, and β_m, respectively ($m = 1$ for the top layer and $m = 2$ for the bottom layer). The effect of distortion of the interface is neglected, and the interface is assumed to be flat and nondeformable ($y = 0$).

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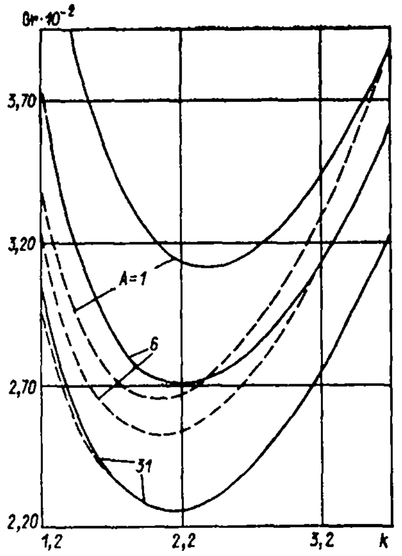

Fig. 1

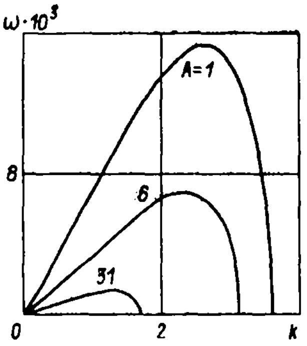

Fig. 2

We assume that a surface-active impurity, characterized by the volume concentration C(x, y) and the surface concentration Γ(x), is dissolved in the lower-lying liquid. With an allowance for convection, diffusion, adsorption, and desorption, the variation of Γ(x) in SAM is described by the equations [2]

$$\frac{\partial\Gamma}{\partial t}+\frac{\partial}{\partial x}(v_x\Gamma)=D_0\frac{\partial^2\Gamma}{\partial x^2}+j,\quad j=-\frac{1}{\tau}(\Gamma-\Gamma_e),$$

where Γ_e is the thermodynamic equilibrium value of the concentration, v_x is the horizontal velocity component of the liquid at the interface, D_0 is the coefficient of surface diffusion of SAM, and τ is the relaxation time. The distribution of C is determined by the boundary-value problem

$$\partial C/\partial t+\mathbf{v}\nabla C=D_C\Delta C,\ -a_2<y<0,$$

$$y=-a_2{:}\ \partial C/\partial y=0,\ y=0{:}\ -D_C\partial C/\partial y=j$$

(D_C is the volume diffusion coefficient of SAM). We assume that the dependence of the surface tension coefficient on the temperature and Γ(x) is linear: $\sigma=\sigma_0-\alpha_T T-\alpha_\Gamma\Gamma$ (σ_0, α_T, and α_Γ are constants).

The mechanical equilibrium is characterized by a constant value of the vertical temperature gradient, which is equal to $-\theta\varkappa_2/(\varkappa_2 a_1+\varkappa_1 a_2)$ in the top medium and $-\theta\varkappa_1/(\varkappa_2 a_1+\varkappa_1 a_2)$ in the lower medium, and also by constant values of the surface Γ_0 and the volume C_0 concentrations of SAM. The equilibrium temperature and pressure at the interface are denoted by T_0 and p_0, respectively.

Let us now investigate the stability of the equilibrium. We introduce the following notation: $\eta=\eta_1/\eta_2$, $\nu=\nu_1/\nu_2$, $\varkappa=\varkappa_1/\varkappa_2$, $\chi=\chi_1/\chi_2$, $\beta=\beta_1/\beta_2$, and $a=a_2/a_1$. We use a_1, a_1^2/ν_1, ν_1, θ, Γ_0, Γ_0/l, and $\Gamma_0\nu_1/la_1$ as the units of length, time, stream function, temperature, surface concentration, volume concentration, and flux density, respectively ($l=(\partial\Gamma/\partial C)_{C_0,T_0,p_0}$). We impose on the equilibrium state perturbations of the stream function Ψ_m', the temperature T_m', the volume concentration C', and the surface concentration Γ':

$$(\psi_1',T_1',\psi_2',T_2',C',\Gamma',j')=(\psi_1(y),$$

$$T_1(y),\psi_2(y),T_2(y),C(y),\Gamma,j)\exp[ikx-(\lambda+i\omega)t].$$

Here, k is the wave number, and $\lambda+i\omega$ is the complex decrement.

The linearized equations for perturbations of the stream function and the temperature have the following form (the dependence of the density of the medium on the impurity concentration is neglected) [1]:

$$(\lambda+i\omega)D\psi_m=-d_mD^2\psi_m+ik\,\mathrm{Gr}\,b_mT_m,$$

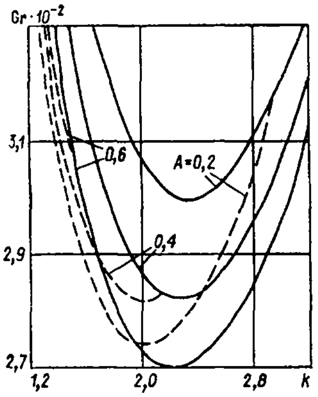

Fig. 3

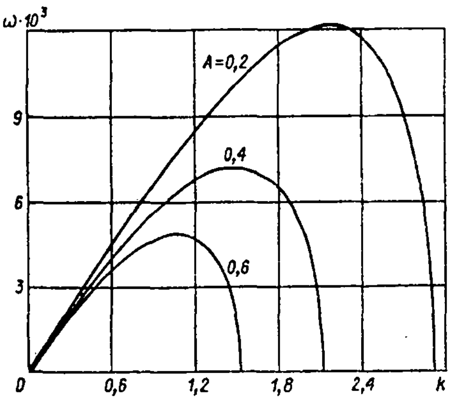

Fig. 4

$$-(\lambda+i\omega)\,T_m-ik\psi_m A_m=\frac{c_m}{\mathrm{Pr}}\,DT_m\quad(m=1,2), \tag{1.1}$$

where $D=d^2/dy^2-k^2$; $b_1=c_1=d_1=1$; $b_2=1/\beta$; $c_2=1/\chi$; $d_2=1/\nu$; $\mathrm{Gr}=g\beta_1\theta a_1^3/\nu_1^2$ is the Grashof number; $\mathrm{Pr}=\nu_1/\chi_1$ is the Prandtl number; $A_1=-1/(1+\varkappa a)$, and $A_2=-\varkappa/(1+\varkappa a)$ are the dimensionless temperature gradients in the top and bottom mediums. Denoting differentiation with respect to y by a prime, we write the conditions at the solid boundaries:

$$y=1:\quad \psi_1=\psi_1'=T_1=0,\quad y=-a:\ \psi_2=\psi_2'=T_2=0. \tag{1.2}$$

As was shown in [2], the conditions at the interface can be reduced to the following form:

$$\begin{gathered}y=0:\quad \psi_1=\psi_2=0,\ \psi_1'=\psi_2',\ T_1=T_2,\ \varkappa T_1'=T_2',\\ \eta\psi_1''+\frac{k^2B\psi_1'}{\lambda+i\omega-D_sk^2-A^2E\,(1+AE\,\mathrm{Sc}\coth\,(qa)/q)^{-1}}=\psi_2''.\end{gathered} \tag{1.3}$$

Here, $B=\alpha_\Gamma\Gamma_0a_1/\eta_2\nu_1$; $D_s=D_0/\nu_1$; $q=(k^2-(\lambda+i\omega)\mathrm{Sc})^{1/2}$; $\mathrm{Sc}=\nu_1/D_C$ is the Schmidt number. For $A^2E=a_1^2/\nu_1\tau\rightarrow0$ (the relaxation time for the dissolving process is long in comparison with the characteristic hydrodynamic time), the solution of SAM does not affect the instability of equilibrium. Conversely, if $A \gg 1$, we arrive at the problem of stability of the system's equilibrium in the absence of SAM. Problem (1.1)-(1.3) describes both the monotonic and the oscillatory branches of instability. The numerical solution of the boundary-value problem was obtained by using the Runge—Kutta method.

We shall first provide the results of the calculations performed for the air—water system with the following parameters: $\mathrm{Pr}=0.758$, $\eta=0.0182$, $\nu=15.1$, $\varkappa=0.0396$, $\chi=138$, $\beta=17.7$, and $a=1$. In the absence of SAM, the instability of equilibrium for this system is monotonic. The calculations were performed for $D_s=10^{-3}$, $B=0.02$, and $E=100$.

Figure 1 shows the monotonic (solid) and oscillatory (dashed) neutral curves for $\mathrm{Sc}=10^3$. It is evident that, with an increase in the solubility parameter A, the monotonic neutral curves descend, narrowing the region where fluctuations occur. The point marking the end of the oscillatory neutral curve is shifted to the long-wave region. If the A_* value is exceeded, the monotonic mode is the most critical one. Figure 2 shows the oscillation frequency ω as a function of the wave number k, which indicates that the oscillation frequency decreases with an increase in A.

Similar diagrams for $\mathrm{Sc}=10^2$ are given in Figs. 3 and 4. Comparison with the calculation results for $\mathrm{Sc}=10^3$ indicates that a reduction in Sc (an increase in the coefficient of volume diffusion) results in more effective suppression of oscillations with an increase in A.

Figure 5 shows the Grashof number, minimized with respect to k, as a function of B ($\mathrm{Sc}=10^2$). The range of the parameter B values where the oscillatory instability is most critical narrows as A increases.

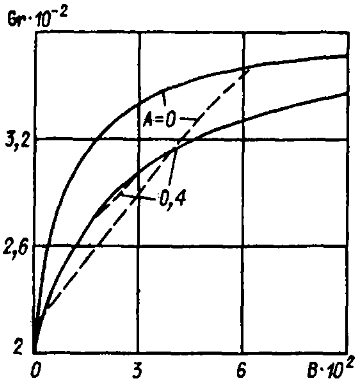

Fig. 5

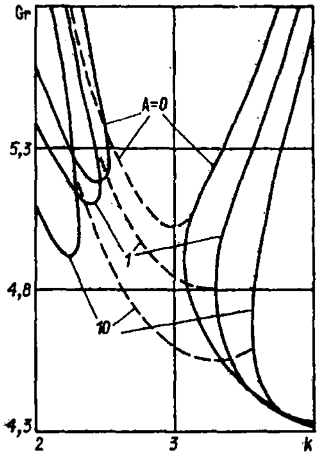

Fig. 6

Consider the transformer oil—formic acid system for Pr = 306.32, η = 11.123, ν = 15.408, $\varkappa = 0.41$, χ = 0.714, β = 0.672, and a = 0.667. The calculations were performed for $D_S = 10^{-3}$, B = 0.05, E = 0.01, and Sc = 100. In the absence of SAM, the neutral curve for this system has an oscillatory segment, which connects the long-wave and the short-wave fragments of the monotonic neutral curves [3] (Fig. 6). With an increase in A, all the neutral curves are shifted toward smaller Gr values as they approach the limiting position corresponding to the absence of SAM.

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